# Note: Robust Regression

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Coefficients are estimated by iteratively reweighted least squares (IRLS). At iteration (t), the steps are:

(1) Compute the weighted least-squares estimate $\beta^{(t)}$ using the weights from the previous iteration $w_i^{(t-1)}$,

$$\beta^{(t)} = \arg\min_{\beta} \sum_{i=1}^{n} w_i^{(t-1)} \big(y_i - x_i^\top \beta\big)^2.$$

The initial weights $w_i^{(0)}$ are all equal to 1.

(2) Compute residuals $r_i^{(t)}$ based on the current coefficients $\beta^{(t)}$,

$$r_i^{(t)} = y_i - x_i^\top \beta^{(t)}.$$

(3) Estimate the scale $s^{(t)}$ of the residuals, using MAD,

$$s^{(t)} = 1.4826 \cdot \text{median}\big(\big|r_i^{(t)} - \text{median}(r_i^{(t)})\big|\big).$$

(4) Standardize the residuals to get $u_i^{(t)}$,

$$u_i^{(t)} = \frac{r_i^{(t)}}{s^{(t)}},$$

(5) Compute new weights $w_i^{(t)}$ using the chosen $\psi$ function applied to the standardized residuals,

$$w_i^{(t)} = \psi(u_i^{(t)}), \quad \text{with} \quad \psi(u) = \min\left(1, \frac{1.345}{|u|}\right).$$