

Perceptron and Sigmoid Neuron

1 Perceptron

A perceptron is a function that maps a real-valued vector to a binary outcome:

$$\begin{aligned} f : \mathbb{R}^n &\rightarrow \{0, 1\} \\ x &\mapsto h(w^T x + b) \end{aligned}$$

where h is the [Heaviside step function](#) and w and b are the weight vector and bias, respectively.

Constructing a NAND gate with a perceptron

The following perceptron defines a NAND gate:

$$\begin{aligned} f : \{0, 1\}^2 &\rightarrow \{0, 1\} \\ x &\mapsto h((-2, -2)^T x + 3) \end{aligned}$$

This perceptron maps the input to the output as follows:

x_1	x_2	$f(x)$
0	0	1
0	1	1
1	0	1
1	1	0

2 Sigmoid Neuron

A sigmoid neuron is a function that maps a real-valued vector to a real-valued output. The sigmoid function is defined as:

$$\begin{aligned} f : \mathbb{R}^n &\rightarrow (0, 1) \\ x &\mapsto \sigma(w^T x + b) \end{aligned}$$

where $\sigma(z) = \frac{1}{1 + e^{-z}}$ is the [sigmoid function](#), and w and b are the weight vector and bias, respectively.