Chapter 2

Exercise 2.1

The cancer screening test has a sensitivity of 0.9 and a specificity of 0.97. We suppose now that the prevalence is 0.001 (was 0.01 in the example). The probability that a person has cancer given that they have a positive test result is

$$\begin{aligned}\Pr(C=1 \mid T=1) &= \frac{\Pr(T=1 \mid C=1)\Pr(C=1)}{\Pr(T=1 \mid C=1)\Pr(C=1) + \Pr(T=1 \mid C=0)\Pr(C=0)} \\ &= \frac{0.9 \cdot 0.001}{0.9 \cdot 0.001 + 0.03 \cdot 0.999} \\ &\approx 0.029.\end{aligned}$$

Exercise 2.2

Let Y, B, G and R be the result of yellow, blue, green and red dice rolls, respectively. The dice rolls are given in the table below.

k	$\Pr(Y=k)$	$\Pr(B=k)$	$\Pr(G=k)$	$\Pr(R=k)$
0	0	1/3	0	0
1	0	0	1/2	0
2	0	0	0	2/3
3	1	0	0	0
4	0	2/3	0	0
5	0	0	1/2	0
6	0	0	0	1/3

We have

$$\begin{aligned}\Pr(Y > R) &= \Pr(Y=3, R=2) = 2/3 \\ \Pr(B > Y) &= \Pr(B=4, Y=1) = 2/3 \\ \Pr(G > B) &= \Pr(G=1, B=0) + \Pr(G=5) = 1/2 \cdot 1/3 + 1/2 = 2/3 \\ \Pr(R > G) &= \Pr(R=2, G=1) + \Pr(R=6) = 2/3 \cdot 1/2 + 1/3 = 2/3,\end{aligned}$$

which shows the *non-transitivity* of the dices.

Exercise 2.3

$$\begin{aligned}p(\boldsymbol{y}) &\overset{(1)}{=} \iint p(\boldsymbol{y} \mid \boldsymbol{u}, \boldsymbol{v})\, p_{\boldsymbol{u}}(\boldsymbol{u})\, p_{\boldsymbol{v}}(\boldsymbol{v})\, d\boldsymbol{u}\, d\boldsymbol{v} \\ &\overset{(2)}{=} \iint \delta(\boldsymbol{y} - \boldsymbol{u} - \boldsymbol{v})\, p_{\boldsymbol{u}}(\boldsymbol{u})\, p_{\boldsymbol{v}}(\boldsymbol{v})\, d\boldsymbol{u}\, d\boldsymbol{v} \\ &\overset{(3)}{=} \iint p_{\boldsymbol{u}}(\boldsymbol{u})\, p_{\boldsymbol{v}}(\boldsymbol{u} - \boldsymbol{v})\, d\boldsymbol{u}\end{aligned}$$

(1) Law of total probability.

(2) The conditional distribution $p(\boldsymbol{y} \mid \boldsymbol{u}, \boldsymbol{v})$ is a Dirac distribution centered at $\boldsymbol{u} + \boldsymbol{v}$.

(3) Integrate over $\boldsymbol{v}$.